**Math 231 – HW 7 Name: Troy Jeffery**

***Remember -- FORMAT is as important as CONTENT – get them both right!***

3.4 18, 25 3.6 1, 2, 9.

***Warm-up Questions -- Negations of Quantified Statements.***

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| *The original quantified statement, in words:*  All dinosaurs like movies. | *The negation of the original statement, in words:*  There is a dinosaur who does not like movies. |
| *The original statement, in symbols:* | *The negation of the original statement, in symbols:* |

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| *The original quantified statement, in words:*  There is a dinosaur who is purple. | *The negation of the original statement, in words:*  No dinosaur is purple. |
| *The original statement, in symbols:* | *The negation of the original statement, in symbols:* |

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| *The original quantified statement, in words:*  The square of any real number is positive. | *The negation of the original statement, in words:*  There is a real number whose square is not positive. |
| *The original statement, in symbols:* | *The negation of the original statement, in symbols:* |

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| *The original quantified statement, in words:*  There is a real number that can be expressed as a fraction (of integers). | *The negation of the original statement, in words:*  No real number can be expressed as a fraction (of integers). |
| *The original statement, in symbols:* | *The negation of the original statement, in symbols:* |

**3.4 (18)** Give a formal proof of the theorem:

Theorem: The product of any two consecutive integers is even.

Proof: Given any two consecutive integers n and n+1, either n is even or n is odd.

Case 1: Assume n is even.

Therefore n = 2k, and n+1 = 2k+1, for some integer k.

So, n(n+1) =

=

=

= (The product and sum of integers is always an integer)

=

Case 2: Assume n is odd.

Therefore, n=2k+1 and n+1=2k+2 for some integer k.

So, n(n+1) =

=

=

= (The product and sum of integers is always an integer)

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**3.4 (25)** Give a formal proof of the theorem:

Theorem: The square of any integer has the form 4m or 4m+1 for some integer m.

Proof: For any integer n, either n is even or n is odd.

Case 1: Suppose n is even.

Therefore n = 2k, for some integer k.

So, n2 =

=

= (The product and sum of integers is always an integer)

=

Case 2: Suppose n is odd.

Therefore n = 2k+1, for some integer k.

So, n2 =

=

=

= (The product and sum of integers is always an integer)

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**3.6 (1)** ***First, let's explore the idea that " There is no positive real number that is smaller than all other positive real numbers."***

Pick any really small positive real number: **1**

Could it possibly be the smallest positive real number? Let's see:

Take half of your number: **.5**

Is it still positive? **Yes**

Is it smaller than your original number? **Yes**

Ok, let's try again -- maybe your new number is the smallest possible real number. Let's check:

Take half of your new number: **.25**

Is it still positive? **Yes**

Is it smaller than your original number? **Yes**

Now, explain why you believe that there is no smallest possible real number:

**Any number can always be divided by two.**

*Let's do the formal proof:*

Theorem: There is no positive real number that is smaller than all other positive real numbers.

Proof: Suppose not. In other words, assume that there is some real number x such that x is positive and for all positive real numbers y,

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| *Extra question: Rewrite the above statement using symbols instead of words:* |

Consider the number .

We know that  is positive because **x < 0**

We know that  because

So, now we know that .

But wait! We assumed that x was **less than** all positive real numbers, and now we've shown that y is **positive** and **less** than x.

Contradiction.

Therefore, there is no smallest positive real number.

**3.6 (2)** Finish the following proof that there is no greatest (largest) positive even integer.

Theorem: There is no greatest even integer.

Proof: Suppose not. In other words, suppose that there is some positive, even integer n such that n is greater than all even integers.

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| *Extra question: Rewrite the above statement using symbols instead of words. Use the variable m for your extra variable.* |

Consider the number .

We know that m is positive because **n > 0.**

We know that m is an integer because **the sum or product of integers is an integer.**

We know that m is even because **2(integer) = integer.**

We know that  because **twice a positive integer is greater than the original.**

So, now we know that **m > n.**

But wait! We assumed that n was **greater than** all positive **even integers** and now we've shown that m is **an even integer** and **greater** than n.

Contradiction.

Therefore, **there is no greatest positive even integer.**

**3.6 (9)** Prove the theorem in two ways -- by contraposition and by contradiction.

Theorem: The negative of any irrational number is irrational.

*By contraposition:*

Exploration: Rewrite the theorem as an if-then conditional:

∀ x∈, if then

Write the contraposition of your if-then statement:

∀ x∈, if

*Now, let's do the proof by contraposition:*

Theorem: The negative of any irrational number is irrational.

Proof: It is sufficient to show that:

(insert your contrapositive statement).

Since -x is rational, then

Therefore, =

= The products and sums of integers are integers.

= rational.

So, contradiction.

*By contradiction:*

Exploration: Write the negation of the theorem:

There is some irrational number whose negative is rational.

*Now, let's do the proof by contradiction:*

Theorem: The negative of any irrational number is irrational.

Proof: Suppose not. In other words there is some irrational number whose negative is rational

(insert your negation of the theorem).

Let x be our irrational number.

Since -x is rational, then -x =

Therefore,

=

= rational which is the opposite of our assumption. So, Contradiction